

#### Foundations of Logic: Overview

- · Propositional logic:
  - Basic definitions.
  - Equivalence rules & derivations.
- Predicate logic:
  - Predicates.
  - Quantified predicate expressions.
  - Equivalences & derivations.

#### Propositional Logic

- Propositional Logic is the logic of compound statements built from simpler statements (atomic propositions) using connectives (Boolean).
- Calculus:
  - A method of reasoning by computation of symbols
- Propositional Calculus:
  - Scheme for calculating with logic formulas
  - Three different Propositional Calculuses:
  - Semantics-based reasoning truth tables
  - Syntax-based reasoning inference rules
  - Equational reasoning Boolean algebra

#### **Propositional Logic**

- The Two Elements of Symbolic Logic:
  - Proposition
  - Logical connectives

#### Two Elements of Propositional Logic: Propositions

#### Propositions:

- A proposition is a statement with a truth value. That is, it is a statement that is true or else a statement that is false. Here are some examples with their truth values.
- A proposition (denoted p, q, r, ...) is simply:
  - a statement (i.e., a declarative sentence) - with some definite meaning, (not vague or ambiguous)
  - having a truth value that's either true (T) or false (F)
  - it is never both, neither, or somewhere "in between!"
    - » However, you might not know the actual truth value, and » The truth value might depend on the situation or context.

#### Two Elements of Propositional Logic: Propositions

- Examples of Propositions:
  - "It is terribly hot outside." (In a given situation.)
  - There are only finitely many prime numbers. (false)
  - "Islamabad is the capital of Pakistan."
  - "4 + 2 = 6"

#### Two Elements of Propositional Logic: Propositions

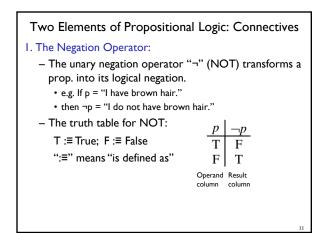
- Examples of NOT Propositions:
  - "Who are you?" (interrogative)
  - "Wah Wah" (meaningless exclamation)
  - "Please stop it!" (imperative)
  - "Where is PIEAS?"
  - "Yes, I want to be there" (vague)
  - "2 + 2" (expression with a non-true/false value)
  - Coffee tastes better than tea. (Again, this is a matter of taste, not truth.)
  - Blue is the best color to paint a house.

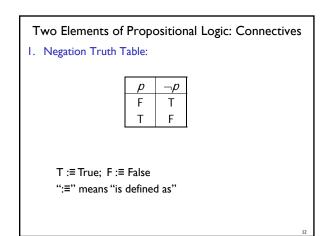
#### Two Elements of Propositional Logic: Connectives

- Connectives (Logical Operators):
  - Arithmetic operators (operations) such as addition, subtraction, multiplication, division, and negation act on numbers to give new numbers.
  - Logical operators act on propositions to give new (compound) propositions.
  - The truth value of the compound proposition depends only on the truth value of the component propositions. Such a list is a called a truth table.

# Two Elements of Propositional Logic: Connectives Connectives (Logical Operators):

Formal Name	Nickname	<u>Arity</u>	Symbol
Negation operator	NOT	Unary	٦
Conjunction operator	AND	Binary	^
Disjunction operator	OR	Binary	V
Exclusive-OR operator	XOR	Binary	$\oplus$
Implication operator	IMPLIES	Binary	$\rightarrow$
Biconditional operator	IFF	Binary	$\leftrightarrow$





#### Two Elements of Propositional Logic: Connectives

- 2. The Conjunction Operator:
  - - e.g. If p="1 will have salad for lunch." and q="1 will have steak for dinner.", then p∧q="1 will have salad for lunch and AND I will have steak for dinner."

# Two Elements of Propositional Logic: Connectives 2. Conjunction Truth Table:

р	q	p∧q
Т	Т	Т
Т	F	F
F	Т	F
F	F	F

- True iff both operands are True
  - $\bullet \ \mbox{Define } P \ \ = x > 0, \ \ \ Q \ \ = x < 10$
  - +  $P \wedge Q \;\; \text{is True iff } x \; \text{is between 0 and 10}$

### Two Elements of Propositional Logic: Connectives

#### 3. The Disjunction Operator:

- The binary disjunction operator "∨" (OR) combines two propositions to form their logical disjunction.
  - p="My car has a bad engine."
  - q="My car has a bad carburetor."
  - pvq="Either my car has a bad engine, or my car has a bad carburetor."

# Two Elements of Propositional Logic: Connectives

#### 3. Disjunction Truth Table:

 $- p \lor q$  means that p is true, or q is true, or both are true. This operation is also called inclusive or, because it includes the possibility that both p and q are true.

p	q	p∨q
Т	Т	Т
Т	F	Т
F	Т	Т
F	F	F

Two Elements of Propositional Logic: Connectives Exercise:

- Let p="lt rained last night", q="The sprinklers came on last night," r="The lawn was wet this morning."
- Translate each of the following into English:

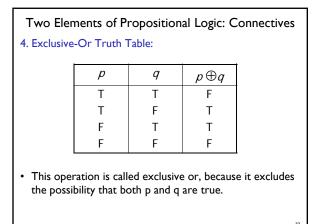
¬p = "It didn't rain last night."

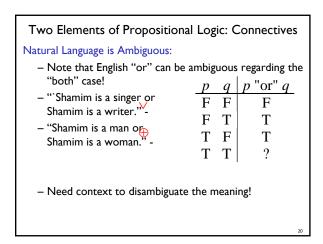
¬ r ∨ p ∨ q = "Either the lawn wasn't wet this morning, or it rained last night, or the sprinklers came on last night."

#### Two Elements of Propositional Logic: Connectives

#### 4. The Exclusive Or Operator:

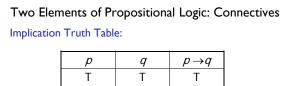
- The binary exclusive-or operator "
   "
   (XOR)
   combines two propositions to form their logical
   "exclusive or" (exjunction?).
  - p = "I will earn an A in this course,"
  - q = "I will drop this course,"
  - $p\oplus q$  = "I will either earn an A in this course, or I will drop it (but not both!)"





#### Two Elements of Propositional Logic: Connectives

- 5. The Implication Operator:
- The implication  $p \rightarrow q$  states that p implies q.
  - i.e., If p is true, then q is true; but if p is not true, then q could be either true or false.
    - e.g., let p = "You study hard."
       q = "You will get a good grade."
  - $-p \rightarrow q$  = "If you study hard, then you will get a good grade." (else, it could go either way)



Т	Т	Т
Т	F	F
F	Т	Т
F	F	Т

•  $p \rightarrow q$  is false only when p is true but q is not true.

#### Conditional: why $F \rightarrow F$ is True

• Mathematically, p should imply q whenever it is possible to derive q by from p by using valid arguments. For example consider the following:

$$-$$
 If 0 = 1 then 3 = 9.

• Q: Is this true mathematically?

#### Conditional: why $F \rightarrow F$ is True

A: YES mathematically and YES by the truth table.
Here's a mathematical proof:

0 = 1 (assumption)

#### Conditional: why $F \rightarrow F$ is True

A: YES mathematically and YES by the truth table. Here's a mathematical proof:

I. 0 = I (assumption)

2. I = 2 (added I to both sides)

#### Conditional: why $F \rightarrow F$ is True

A: YES mathematically and YES by the truth table.

Here's a mathematical proof:

- I. 0 = I (assumption) 2. I = 2 (added I to both sides)
- 3. 3 = 6 (multiplied both sides by 3)

#### Conditional: why $F \rightarrow F$ is True

A: YES mathematically and YES by the truth table.

## Here's a mathematical proof:

- I. 0 = I (assumption)
- 2. I = 2 (added I to both sides)
- 3. 3 = 6 (multiplied both sides by 3)
- 4. 0 = 3 (multiplied no. 1 by 3)

# Conditional: why $F \rightarrow F$ is True

A: YES mathematically and YES by the truth table. Here's a mathematical proof:

- I. 0 = I (assumption)
- 2. I = 2 (added I to both sides)
- 3. 3 = 6 (multiplied both sides by 3)
- 4. 0 = 3 (multiplied no. I by 3)
- 5. 3 = 9 (added no. 3 and no. 4)

# Two Elements of Propositional Logic: Connectives

Examples of Implications:

- "If this lecture ever ends, then the sun will rise tomorrow."(True or False?
- "If Tuesday is a day of the week, then I am a penguin." True or False?
- "If I+I=6, then Bush is president." True or False?
- "If the moon is made of green cheese, then I am richer than Bill Gates." True or False?

# Two Elements of Propositional Logic: Connectives

English Phrases Meaning  $p \rightarrow q$ 

- "p implies q"
- "if p, then q"
- "if p, q"
- "when p, q"
- "whenever p, q"
- "q if p"
- "q when p"
- "q whenever p"
- "p is sufficient for q"

• "p only if q"

• "q is necessary for p"

QED

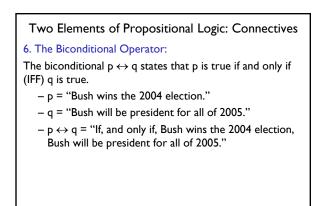
- "q follows from p"
- "q is implied by p"

### Two Elements of Propositional Logic: Connectives

Converse, Inverse, Contrapositive:

- $\bullet \ \ \, \text{For an implication} \qquad p \to q \text{:}$ 
  - $\text{ Its converse is:} \qquad q \to p.$
  - $\text{ Its inverse is:} \qquad \neg p \to \neg q.$
  - Its contrapositive:  $\neg q \rightarrow \neg p$ .
  - One of these three has the same meaning (same truth table) as  $p\to q.\,$  Can you figure out which?

<ul> <li>Two Elements of Propositional Logic: Connectives</li> <li>Proving the equivalence of p → q and its contrapositive using truth tables:</li> </ul>								
-	<u>р</u> F F T T	q F T F T	$\neg q$ F T F	<i>¬p</i> T T F F	$\begin{array}{c} p \rightarrow q \\ T \\ T \\ F \\ T \end{array}$	$ \begin{array}{c} \neg q \rightarrow \neg p \\ T \\ T \\ F \\ T \\ \end{array} $		
							32	



# Two Elements of Propositional Logic: Connectives

- Biconditional Truth Table:
- $p \leftrightarrow q$  means that p and q have the same truth value.

p	q	$p \leftrightarrow q$
Т	Т	Т
Т	F	F
F	Т	F
F	F	Т

•  $p \leftrightarrow q$  does not imply that p and q are true, or that either of them causes the other, or that they have a common cause.

Two Elements of Propositional Logic: Connectives <ul> <li>Boolean Connectives Summary:</li> </ul>							
<u>р</u> F F F T T F T T	T	<i>p∧q</i> F F F T	Т	<i>p⊕q</i> F T T F	$\begin{array}{c} p \rightarrow q \\ T \\ T \\ F \\ T \end{array}$	$\begin{array}{c} p \leftrightarrow q \\ T \\ F \\ F \\ T \end{array}$	

# Two Elements of Propositional Logic: Connectives

Some Alternative Notations:

Name:	not	and	or	xor	implies	iff
Propositional logic:	Γ	^	$\vee$	$\oplus$	$\rightarrow$	$\leftrightarrow$
Boolean algebra:	$\overline{p}$	pq	+	$\oplus$		
C/C++/Java (wordwise):	!	&&		! =		==
C/C++/Java (bitwise):	~	&		^		
Logic gates:	$\diamondsuit$	$\square$	À	$\square$		

#### Bits and Bit Operations

- A bit is a binary (base 2) digit: 0 or 1.
- Bits may be used to represent truth values.
- By convention: 0 represents "false"; I represents "true".
  Boolean algebra is like ordinary algebra except that
- variables stand for bits, + means "or", and multiplication means "and"

#### **Bitwise Operations**

- Boolean operations can be extended to operate on bit strings as well as single bits.
  - e.g.: 01 1011 0110 11 0001 1101 11 1011 1111 Bit-wise OR 01 0001 0100 Bit-wise AND 10 1010 1011 Bit-wise XOR

#### Recap

- You have learned about: Next lecture:
- Propositions: What they are? Propositional
- Propositional logic operators
  - rs equivalences. – How to prove them.
  - Symbolic notations.English equivalents.
  - Logical meaning.
- Truth tables.
- Atomic vs. Compound propositions.
- Alternative notations.
- Bits and bit-strings.